You guessed it: 2D Lattice Gas with Phase Transition

Consider a two-dimensional (2D) square lattice consisting of N sites. Each site can be either occupied ($n_i = 1$) or empty ($n_i = 0$). Particles in neighboring sites interact attractively. We will model this attraction as a constant negative energy $-\epsilon$. Moreover, a filled site contributes an energy $-\mu$ to the total energy of the system. Do not make assumptions yet on the sign of μ. The Hamiltonian of the system is given by

$$H = -\epsilon \sum_{\langle i,j \rangle} n_i n_j - \mu \sum_i n_i,$$

where the first sum runs over nearest neighbor pairs. In what follows we will call the average number of particles per site ($\langle n_i \rangle$) the density of the system. Assume that the lattice is under canonical conditions.

(a) Discuss qualitatively (i.e. without necessarily writing any equation) the behavior of the density at high temperature. (2p)

(b) Discuss the behavior of the density at low temperature as a function of μ. (2p)

(c) Assuming that the fluctuations in density are very small and that the system is translationally invariant (say, by a distance equal to the lattice constant), derive a mean field expression for the Hamiltonian and show that it is equal to

$$H_{\mathrm{MF}} = -4\epsilon\rho \sum_{i=1}^{N} n_i + 2\epsilon\rho^2 N - \mu \sum_i n_i,$$

or, equivalently, for a single site,

$$h_{\mathrm{MF}} = -4\epsilon\rho\, n_i + 2\epsilon\rho^2 n_i - \mu n_i,$$

where $\rho \equiv \langle n_i \rangle$ for simplicity. It might help here to write $n_i = \rho + \delta n_i$. (2p)

(d) Compute the large temperature limit of the density ρ. (1p)

(e) In certain conditions, the high temperature behavior of ρ from the previous point will be reached at a finite temperature T_c. This is called a continuous phase transition. Find the value of μ for which the system has this property. Find the critical temperature T_c. *Hint: the order parameter here is the density ρ.* (3p)

The angular momentum stored in a combination of steady electric and magnetic fields

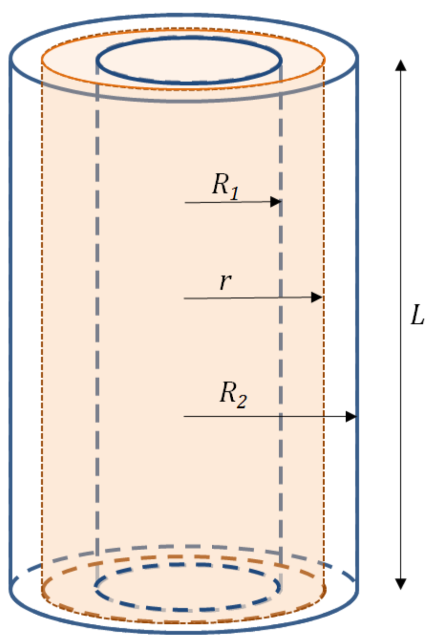
The surface current density of a very long (you can consider it infinitely long) hollow superconducting cylinder of radius r produces through its cross section a steady magnetic flux ϕ. The superconducting cylinder is coaxial with two cylindrical surfaces of length L and radii $R_1 < r < R_2$ having electric charge densities σ and $-\sigma$, respectively.

a) Calculate the angular momentum stored in the electric and magnetic fields of this system.

b) If the temperature is gradually raised until the superconducting state is destroyed, will the two charge distributions rotate? If so, predict the direction of rotation with respect to the initial direction of the supercurrent.

c) What angular momentum will this system have after the superconducting state is destroyed?

Note: You will consider that in the initial state the superconducting cylinder and the electrically charged surfaces are at mechanical rest



Electrostatics of a Cosmic String

A *cosmic string* is a one-dimensional object with an extraordinarily large linear mass density ($\mu \sim 10^{22}$ kg/m) which, in some theories, formed during the initial cool-down of the Universe after the Big Bang. In two-dimensional (2D) general relativity, such an object distorts flat space-time into an extremely shallow cone with the cosmic string at its apex. Alternatively, one can regard flat 2D space as shown below: undistorted but with a tiny wedge-shaped region removed from the physical domain. The usual angular range

$$0 < \phi < 2\pi$$

is thus reduced to

$$0 < \phi < \frac{2\pi}{p},$$

where

$$p^{-1} = 1 - \frac{4G\mu}{c^2},$$

with G being Newton's gravitational constant and c the speed of light. The two edges of the wedge are indistinguishable so that any physical quantity $f(\phi)$ satisfies

$$f(0) = f\left(\frac{2\pi}{p}\right).$$

1. Free-Space Green Function in 2D

Begin with no string. Show that the free-space Green function in 2D is

$$G_0(\vec{r}, \vec{r}\,') = -\frac{1}{2\pi\epsilon_0} \ln |\vec{r} - \vec{r}\,'|,$$

where $G_0(\vec{r}, \vec{r}\,')$ represents the electric potential at $\vec{r}$ due to a unit line charge at $\vec{r}\,'$.

2. Delta Function Representation

Now add the string so that $p \neq 1$. To find the modified free-space Green function $G_{p0}(\vec{r}, \vec{r}\,')$, a representation of the delta function which exhibits the proper angular behavior is required. Show that a suitable form is

$$\delta(\phi - \phi') = \frac{p}{2\pi} \sum_{m=-\infty}^{+\infty} e^{imp(\phi - \phi')}.$$

3. Ansatz and Mode Expansion

Exploit the ansatz

$$G_{p0}(\rho,\phi,\rho',\phi') = \frac{p}{2\pi}\sum_{m=-\infty}^{+\infty} e^{imp(\phi-\phi')}\, G_m(\rho,\rho')$$

to show that

$$G_{p0}(\rho,\phi,\rho',\phi') = \frac{1}{2\pi}\sum_{m=1}^{\infty}\cos\Big[mp(\phi-\phi')\Big]\frac{1}{m}\left(\frac{\rho_<}{\rho_>}\right)^{mp} - \frac{p}{2\pi}\ln\rho >,$$

where

$$\rho_> = \max(\rho,\rho') \quad \text{and} \quad \rho_< = \min(\rho,\rho').$$

Hint: The 2D Laplace operator takes the form

$$\nabla^2\psi(\vec{r}) = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\psi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\phi^2},$$

and the 2D delta function is given by

$$\delta^{(2)}(\vec{r}-\vec{r}\,') = \frac{\delta(\rho-\rho')\delta(\phi-\phi')}{\rho}.$$

4. Closed-Form Expression for G_{p0}

Argue by inspection that G_{10} and G_{p0} are linked by a simple change of variables. Thus, find a closed-form expression for

$$G_{p0}(\rho,\phi,\rho',\phi'),$$

and check that $G_{10}(\vec{r},\vec{r}\,')$ correctly reproduces your answer in part (1).

5. Force on a Line Charge

Finally, consider a cosmic string at the origin and a line charge q at $\vec{r}$. Think of the cosmic string scenario as being analogous to the presence of an oddly shaped conductor: the Green function contains both a direct contribution from the charge q and an indirect term due to the charge induced on the conductor. In this case, the force felt by the charge q may be computed from the indirect term as

$$\vec{F} = -q^2\nabla\Big[G_{p0}(\vec{r},\vec{r}\,') - G_{10}(\vec{r},\vec{r}\,')\Big]\Big|_{\vec{r}=\vec{r}\,'}.$$

Show that a cosmic string at the origin and a line charge q at $\vec{r}$ are attracted with a force

$$\vec{F} = \frac{(p-1)q^2}{4\pi\epsilon_0\rho}\,\hat{\rho}.$$

Michelson Interferometry

A Michelson interferometer is set up to give circular fringes and is illuminated with light in the spectral range 800–900 nm by a caesium light source. The intensity $I(t)$ at the centre of the interference pattern is recorded as a function of the distance t of the moving mirror from that corresponding to zero path difference. It is found to have the form

$$I(t) = I_0 \left[3 - 3\cos(K_1 t)\cos(K_2 t) + \sin(K_1 t)\sin(K_2 t)\right],$$

where

$$K_1 = 1.439 \times 10^5\ \mathrm{cm}^{-1}, \quad K_2 = 3.48 \times 10^3\ \mathrm{cm}^{-1},$$

and I_0 is a constant.

I. Show that the above expression can be written as the sum of the patterns due to two monochromatic spectral components. Hence determine, for the two caesium atomic lines in this wavelength range:

(a) their mean wavelength, $\bar{\lambda}$,

(b) their wavelength separation, $\Delta\lambda$,

(c) their relative intensities.

(4 points)

II.

(a) Make a sketch of the interference pattern over the range $t \in [-10\,\mu\mathrm{m},\ 10\,\mu\mathrm{m}]$.

(b) Compute the visibility of the fringes and plot this as a function of t over the same range.

(3 points)

III. When the interference pattern is recorded over a much larger range of path difference, it is found that the periodic terms in the pattern are in fact multiplied by a function

$$I(t) = I_0 \left[3 - 3\cos(K_1 t)\cos(K_2 t) + \sin(K_1 t)\sin(K_2 t)\right] \exp\left(-K_3 t^2\right).$$

Suggest what might cause this effect.

Heat capacities

a) For one mole of a van der Waals gas, show that

$$C_p - C_V = \left[p + \left(\frac{\partial U}{\partial V}\right)_T\right]\left(\frac{\partial V}{\partial T}\right)_p,$$

where C_p and C_V are the heat capacities at constant pressure and constant volume respectively, and U and V are the internal energy and volume.

b) Using the above equality and the thermodynamic relation

$$p + \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V,$$

derive an expression for $C_p - C_V$ in terms of the state variables T, V, and p for one mole of a Van der Waals gas.

Fizică nucleară

1)Analiza nucleului de Hafniu

Spinul și paritatea, j^π, precum și energia E pentru starea fundamentală și o secvență de stări excitate ale nucleului de $^{170}_{72}Hf$ sunt date în tabelul următor:

j^π	0^*	2^*	4^*	6^*	8^*
E [keV]	0	100	321	641	1041

- Ce fel de niveluri/nivele de energie sunt?
- Să se calculeze momentul de inerție al nucleului în cel puțin două stări excitate. Să se compare valorile obținute cu cele calculate în ipoteza că nucleul este o sferă rigidă care se rotește. Momentul de inerție al sferei este $I_{sf} = \frac{2}{5}MR^2$. Parametrul r_0 din expresia razei nucleare se va lua egal cu 1.3 fm. Să se comenteze rezultatul.

2)Detectorul Davis

Detectorul Davis folosit pentru studiul dezintegrării β și punerii în evidență prin metode directe a existenței neutrino-ului conținea 600 tone de C_2Cl_4. Clorul conținea 24.47% din izotopul de $^{37}_{17}Cl$. Secțiunea eficace pentru reacția

$$^{37}_{17}Cl \ + \nu_e \ \longrightarrow \ e^- \ + \ ^{37}_{18}Ar,$$

mediată pe partea de peste pragul de producere a spectrului neutrinilor obținuți din dezintegrarea

$$^{8}_{5}B \ \longrightarrow \ ^{8}_{4}Be \ + \ e^- \ + \ \nu_e,$$

este de $10^{-46}\,\mathrm{m}^2$. Producerea de izotopi de $^{37}_{18}Ar$ este de un izotop la 2 zile. Să se calculeze rata de captură și fluxul neutrinilor detectabili.

Se dă masa moleculară a C_2Cl_4, anume: 165.83.

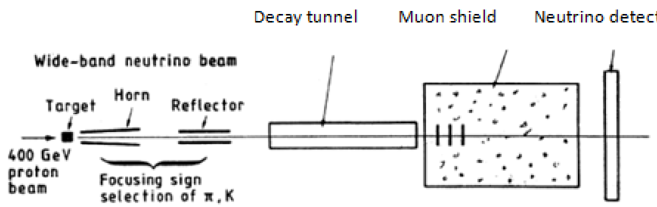
3)„Mezonul” μ

Descoperirea miuonului a introdus, la momentul inițial, o ușoară confuzie cu cuanta de schimb a interacțiilor tari la nivel nuclear, datorită valorii masei de repaus. Ulterior, după descoperirea pionului, în anul 1947, de Powell și colaboratorii, lucrurile s-au clarificat, miuonul devenind extrem de important pentru studiul interacțiilor slabe și fundamentarea Modelului Standard, dar și pentru investigarea proceselor cosmologice fundamentale.

Studiind dezintegrarea mezonului μ prin metoda coincidențelor întârziate, în intervalul de timp $[0 - 2 \times 10^{-6}\,\mathrm{s}]$ au fost înregistrate 200 de dezintegrații, iar în intervalul $[0 - 6 \times 10^{-6}\,\mathrm{s}]$ au fost înregistrate 310 de dezintegrații. Presupunând că dezintegrarea respectă legea dezintegrării radioactive, să se calculeze timpul de viață pentru acest lepton.

Neutrino beam analysis

Neutrino beams have to be produced as secondary beams, because no direct, strongly focused, high-energy neutrino source is available. A schematic layout of a typical neutrino beam line is shown in the next figure:



A proton synchrotron delivers bunches of high-energy protons (of the order of 10^{13} protons or more per bunch) on a fixed target (therefore, the commonly used luminosity units of protons on target – "POT"), resulting in a high yield of secondary mesons, dominantly pions and kaons. By using beam optical devices (dipole or quadrupole magnets or magnetic horns) secondaries of a certain charge sign are focused into a long decay tunnel. There, the secondaries decay mostly via

$$M^+ \to \mu^+ + \nu_\mu \quad ,$$

(assuming focusing of positive secondaries) with a branching ratio of almost 100% for pions and 63.56% for kaons, where M can be either π or K.

Only a fraction of the produced mesons decays in the tunnel with length L_D.

Questions:

1. What is the equation for the decay length for mesons (pions and kaons)? Express the formula as a function of the meson momentum p_M.
2. For $p_M = 200$ GeV, if the probability of survival P is

   $$P = 0.026 \quad \text{(pions)} \quad \text{and} \quad P = 0.181 \quad \text{(kaons)},$$

   determine how long the decay tunnel must be. What is the role of the muon shield?
3. Determine the maximum neutrino energy obtained from pions and kaons as a function of the energy of the meson. The two extreme values are given for $\cos\theta^*_{\max} = \pm 1$ (quantities in the rest frame are marked with an asterisk).

4. Using the transformations

$$E_\nu = \bar{\gamma} E_\nu^* (1 + \bar{\beta} \cos\theta_\nu^*),$$

$$cos\theta_\nu = \frac{cos\theta_\nu^* + \bar{\beta}}{1 + \bar{\beta} cos\theta_\nu^*}$$

determine the angular dependence of the neutrino energy in the laboratory frame.

Supplementary Constants:

- **Pion:** mass = 139 MeV/c^2; mean lifetime = 2.60×10^{-8} s.
- **Kaon:** mass = 494 MeV/c^2; mean lifetime = 1.24×10^{-8} s.
- **Muon:** mass = 105 MeV/c^2.

Evaluation:

(a) 2.5 points

(b) 2 points

(c) 2.5 points

(d) 3 points

Total: $2.5 + 2 + 2.5 + 3 = 10$ points.

Rocket launch

A rocket is launched from the surface of the Earth with an initial velocity v_0 at an angle α with the vertical. Let m be the mass of the rocket, M and R the mass and the radius of the Earth, respectively, and γ the gravitational constant.

(a) (2 points)
Obtain an equation from which to determine the maximum height reached by the rocket. Find the condition for this equation to have solutions and discuss them.

(b) (1 point)
Solve the equation obtained in (a) in the case of a vertical launch ($\alpha = 0°$) with velocity much smaller than $\sqrt{\gamma M/R}$.

(c) (1 point)
If the rocket is launched from the surface of the Earth at an angle $\alpha = 30°$ with the vertical and with initial velocity

$$v_0 = \sqrt{\frac{5}{4}\frac{\gamma M}{R}},$$

find the maximum height achieved by the rocket.

(d) (1 point)
Determine if the rocket launched under the conditions of point (c) may become an Earth satellite.

(e) (2 points)
Describe as precisely as possible the trajectory of the rocket in the conditions of point (c).

(f) (2 points)
What maneuver does the rocket need to perform to become an Earth satellite on a circular orbit of radius equal to the maximum height achieved under the conditions of point (c)?

Air friction, rotation of the Earth, and the influence of the Sun, Moon, and other planets are neglected.

Note: 1 point is awarded by default.
Total: 10 points.