ELECTRICITY and MAGNETISM

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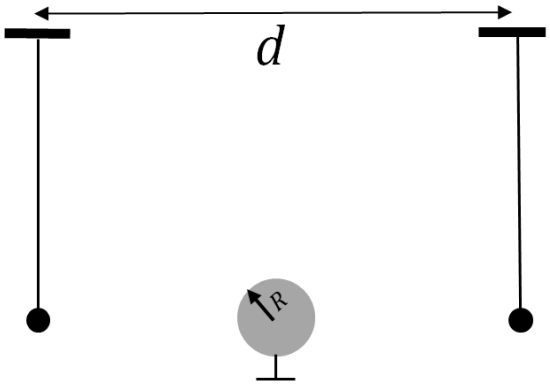

Two identical pendulums consist of negligible sized beads, with the same mass and electric charge. When a grounded conducting sphere is symmetrically interposed in between, they return to a vertical position, with the two beads being located on the horizontal direction passing through the center of the sphere, at distance *d* apart.

What is the *minimum radius* R of the metallic sphere?

The influence of the insulating suspension wires is negligible.

PLANCKS Concurs

Nuclear and Particle Physics

II.1. Semiempirical mass formula has been modified for considering the eccentricity of the nuclear shape inclusion. In this case, for a given nucleus, ${}^{A}_{Z}X_{N}$ *, the binding energy can be wrote as follows:*

$$E_{leg}(A,Z)=\alpha_V A-\beta_S A^{2/3}\left(1+\frac{2}{5}\varepsilon^2\right)-\gamma_C\frac{Z^2}{A^{1/3}}\left(1-\frac{1}{5}\varepsilon^2\right),$$

where ε is the eccentricity.
The values of the coefficients from this formula are the following: $\alpha_V=14\,MeV$, $\beta_S=13\,MeV$, $\gamma_C=0{,}6\,MeV$.
(i) Give physical motivations for this modification
(ii) Calculate the limiting conditions for the mass number, A, and atomic number, Z, for spontaneous nuclear fission of the given nucleus, ${}^{A}_{Z}X_{N}$
(iii) Considering the previous results, indicate if the ${}^{240}_{94}Pu$ *nucleus can present spontaneous nuclear fission*

II.2. In the positron emission tomography, the $\beta^{+¿¿}$ *radioactive isotopes with short lifetimes are very important. There is the hypothesis that an* ${}^{57}_{29}Cu_{28}$ *isotope can exist presenting* $\beta^{+¿¿}$ *decay.*
(i) Motivate the possibility that this nucleus presents $\beta^{+¿¿}$*decay*
(ii) Write the decay equation
(iii) Indicate if the daughter nucleus from the previous decay exists in nature
(iv) Propose an experiment using existing stable isotope ${}^{58}_{28}¿_{30}$ *for obtaining of the* $\beta^{+¿¿}$ *radioactive isotope* ${}^{57}_{29}Cu_{28}$ *. Do the necessary calculations.*

II.3. An accelerator system can offer proton beams, as well as different mesons beams. The detection is done with a bubble chamber.
(a) Present the principle of operation of the bubble chamber
(b) Using the conservation laws and the properties of the elementary particles indicate if are possible the following reactions:
(i) $K^- + p \rightarrow \Omega^- + K^+ + K^0$
(ii) $p + p \rightarrow n + \Delta^{++} + p + \bar{p}$
(iii) $\pi^- + p \rightarrow \sum^+ + K^-$
(c) In a given experiment, a negative pion beam interactions with the sensitive volume of a liquid hydrogen bubble chamber are investigated. In one of the pictures taken during the experiment, two track pairs for charged particles have been observed, connected in two distinct decay vertexes. This type of vertex is called V^0*. Both vertexes are linked with the initial interaction vertex. The distances among the interaction vertex and the two decay vertexes are 11 cm and 37 cm, respectively. For the first* V^0 *vertex, the momentum of the positive charged particle is 0,75 GeV/c, and those of the negative charged particle is 0,25 GeV/c. The angle between the tangents at the two trajectories is of* 22^0*. The corresponding values for the second* V^0

vertex are: 0,4 GeV/c, 1,9 GeV/c, $24^0 30'$, respectively. Both measurement sets have been done in the same conditions. The negative pion beam has a 12 GeV/c momentum.
Solve the following:
- write a general form of the interaction in the sensitive volume of the bubble chamber;
- calculate the rest masses of the two neutral particles decaying in the sensitive volume of the bubble chamber;
- calculate the lifetimes of the two neutral particles;
- identify the two neutral particles.

A simplified version of a polarimeter

A beam of light propagates in the Oz direction, has intensity I_o and is partially elliptically polarized. To determine the degree of polarization of the beam, the following experiment is performed: a. when the beam passes through an ideal linear polarization analyzer, it is found that the intensity of the emerging beam is maximum when the transmission axis of the analyzer is along the Oy direction and minimum when it is along the Ox direction. The ratio of these intensities is I_{max}:I_{min}=2:1. b. The initial beam first passes through a quarter-wave plate, placed with the optical axis along the Oy direction, and then through the ideal linear polarization analyzer. In this case, the maximum intensity is obtained when the transmission axis of the analyzer forms an angle $\theta=60^o$ with the Ox axis.

Calculate the degree of polarization of the original beam ($P=I_{pol}/I_o$)

This experiment is a simplified version of a polarimeter. By combining a quarter-wave plate and a linear analyzer, one can determine the Stokes parameters of a beam and fully characterize its polarization state. Such characterization is, for example, essential in laser-atom interactions, where the polarization determines the angular momentum selection rules and therefore which electronic transitions (e.g., $\Delta m=0,\pm1$) are driven.

Magnetic interactions: an exchange model on 2-lattice sites

The model is described by the Hamiltonian:

$$\hat{H} = t \sum_{\alpha} (c_{1\alpha}^{\dagger} c_{2\alpha} + c_{2\alpha}^{\dagger} c_{1\alpha}) - \frac{J}{2} \sum_{l\alpha\beta} \vec{S}_l \cdot c_{l\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{l\beta}, \tag{1}$$

where $l = 1, 2$ indexes lattice sites, α and β index electron spin states $|\uparrow\rangle$ and $|\downarrow\rangle$, $\vec{S}_{1,2}$ are core spins on lattice sites 1 and 2 ($S \gg 1$), t is the hopping integral of a conduction electron between sites. The second term in the Hamiltonian describes the ferromagnetic exchange interaction of the conduction electron with the core spins, $\vec{\sigma}$ representing Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2}$$

Consider a single electron in this problem ($\sum_{l\alpha} c_{l\alpha}^{\dagger} c_{l\alpha} = 1$).

1. Calculate the lowest energy $\epsilon(S_1, S_2)$ of the electron.[1]

2. Find the form of an effective Hamiltonian H_{eff} describing the interaction of the core spins, using $\epsilon(S_1, S_2)$.[2]

3. What type of magnetic interaction describes H_{eff}, a ferromagnetic or an antiferromagnetic one? Can it be cast in the form of Heisenberg Hamiltonian?

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[1] Hint: Use explicit expressions of Pauli matrices and put the Hamiltonian in a matrix form with respect to the set of annihilation operators $(c_{1\uparrow}, c_{1\downarrow}, c_{2\uparrow}, c_{2\downarrow})$ and the same set of fermion creation operators.

[2] Hint: consider adiabatic approximation. Why is it justified to use it?

Thermodynamics

Problem

A material is brought from temperature T_i to temperature T_f by placing it in contact with a series of N reservoirs at temperatures $T_i+\Delta T$, $T_i+2\Delta T$, …, $T_i+N\Delta T=T_f$. Assuming that the heat capacity of the material, C, does not depend on temperature, calculate the entropy change of the total system, material plus reservoir. What is the entropy change in the limit $N \rightarrow \infty$, for fixed T_f-T_i?

Conformal transformations in electrostatics

1. Consider the transformation defined on $\mathbb{R}^3 - \{\vec{0}\}$ by the rule

$$\vec{r} \to \vec{r}\,' = \frac{R^2}{r^2}\vec{r}\,,$$

where $r = |\vec{r}|$ and R is a constant with dimensions of length. Show that this transformation preserves the angle between two arbitrary position vectors, but not their lengths.

2. Show that if we have a harmonic potential function $\phi(\vec{r})$ with $\nabla^2\phi(\vec{r}) = 0$, then the potential defined as

$$\tilde{\phi}(\vec{r}) = \frac{R}{r} \cdot \phi\left(\frac{R^2}{r^2}\vec{r}\right)$$

is also harmonic, i.e. that $\nabla^2\tilde{\phi}(\vec{r}) = 0$.

3. Show that if $\phi(\vec{r})$ is the potential function corresponding to the charge density $\rho(\vec{r})$, i.e. $\nabla^2\phi(\vec{r}) = -\rho(\vec{r})/\epsilon_0$, then the charge density corresponding to the above potential $\tilde{\phi}(\vec{r})$ is

$$\tilde{\rho}(\vec{r}) = \frac{R^5}{r^5} \cdot \rho\left(\frac{R^2}{r^2}\vec{r}\right)\ .$$

4. Consider the following potentials:

   - $\phi_0(\vec{r}) = Q = \text{constant}$;
   - $\phi_1(\vec{r}) = \vec{p}\cdot\vec{r}$;
   - $\phi_2(\vec{r}) = \sum_{i,j} M_{ij}x_ix_j$.

   Establish first if they are harmonic. Find the corresponding potentials $\tilde{\phi}_k(\vec{r})$ for $k = 0, 1, 2$ and interpret the results.

5. Consider a surface charge density $\sigma(\theta, \varphi)$ on a sphere of radius R. Show that $\sigma(\vec{r})$ and the corresponding potential $\phi_\sigma(\vec{r})$ are invariant with respect to the conformal transformation with parameter R, i.e. that $\tilde{\sigma}(\vec{r}) = \sigma(\vec{r})$ and $\tilde{\phi}_\sigma(\vec{r}) = \phi_\sigma(\vec{r})$. Use the latter to relate the values of the potential ϕ_σ at two points, one inside and one outside of the sphere.

6. Consider a **conducting** sphere of radius R at a fixed potential V_0 and a point charge q situated at $\vec{r}_0$, with $r_0 > R$. Denote as $\phi_q(\vec{r} - \vec{r}_0)$ the Coulomb potential due to q.

   (a) relate $\phi_\sigma(\vec{r})$, $\phi_q(\vec{r} - \vec{r}_0)$ and V_0 for $|\vec{r}| < R$.

   (b) Using also the result from point 5. find the total potential $\phi(\vec{r})$ for $|\vec{r}| > R$. Give the physical interpretation of all terms in its expression.

Note: even if you are unable to solve one or more tasks, you may use the given results to solve the other tasks.

Deriving the Child-Langmuir law

Consider the one-dimensional motion of electrons (charge $-e$ and mass m) in vacuum, from $x = 0$ to $x = \ell$ in the presence of a non-uniform electric field $E(x)$. Suppose the electrons are born at $x = 0$ with an initial velocity $v(x = 0) \simeq 0$. The fixed boundary values of the electric potential are $\phi(x = 0) = 0$ and $\phi(x = \ell) = V$.

1. Find the expression of the local electron velocity $v(x)$ in terms of the local electric potential $\phi(x)$, e and m.

2. For a steady state situation, give the expression of the constant electric current $I > 0$ in terms of the local charge density $\rho(x) < 0$, the local electron velocity $v(x) > 0$ and the cross sectional area A of the electron beam.

   Using the result from the previous point, find the expression of the local charge density $\rho(x)$ in terms of $\phi(x)$ and I, A, e, m.

3. Using the Poisson equation relating the electric potential and the charge density and the previous point's result, obtain a differential equation for $\phi(x)$. Introduce the dimensionless variables $\varphi(x) = \phi(x)/V$ and $\xi = x/\ell$. Show that the equation may be written as

   $$\frac{d^2\varphi}{d\xi^2} = \frac{R}{4}\frac{1}{\sqrt{\varphi(x)}} \ .$$

   Find the constant R in terms of $\epsilon_0, m, e, A, \ell, I, V$.

4. Interpret the previous equation as describing the dynamics of a particle in one-dimensional motion: φ is the position, ξ is the time and the right hand side gives the position-dependent force acting on the particle. Find the corresponding potential energy $U(\varphi)$ for this force field. Write down the energy conservation condition for the fictitious particle and relate its total energy to the electric field $E(x = 0)$.

5. For the case of $E(x = 0) = 0$, determine $\varphi(\xi)$. Plot the corresponding $\phi(x)$ and $E(x)$. From $\varphi(1) = 1$, show that

   $$I = K\, V^{3/2}$$

   and find K in terms of $\epsilon_0, m, e, A, \ell$.

Problems for Plancks's concurs

Academic year 2025-2026

Relativistic and Ultrarelativistic Nuclear Physics

First option

*I.1. Since July 2000 at the **R**elativistic **H**eavy **I**on **C**ollider (RHIC) from **B**rookhaven **N**ational **L**aboratory (BNL) have been performing nucleus-nucleus collisions at ultrarelativistic energies. The main experimental results have been obtained for Au-Au collisions, p-p collisions and d-Au collisions.*

(a) Calculate the rapidity for these collisions at the maximum available energy in the center of mass system, namely $\sqrt{s_{NN}} = 200\,GeV$. *Comment on the results*

(b) For proton-proton collisions at ultrarelativistic energies (E ≅ p, in the system of natural units), demonstrate that the rapidity can be aproximated with the pseudorapidity ($\eta = -\ln\left(tg\frac{\theta}{2}\right)$, *where θ is the scattering angle.*

(c) In the overlaping region of the two colliding nuclei a very dense and hot region is formed. The usual name is fireball. Here can appear new phase of the nuclear matter. One of these is the resonance matter (a phase of nuclear matter with a large number of baryonic resonsnces).

The first baryonic resonance has been discovered by Enrico Fermi and his coworkes, in 1948, and the name was Δ resonance. The $\Delta^{++¿¿}$ *resonance has a mass of 1232 MeV/c² and a full width of* $\Gamma = 120\,MeV$. *The following decay process could appear:* $\Delta^{++¿ \to X + \pi^{+¿¿}¿}$. *Determine the following properties of X:*

- *(i) electric charge*
- *(ii) baryon number*
- *(iii) strangeness number*
- *(iv) if this particle is boson or fermion*
- *(v) lower limit on its mass in MeV/c²*
- *(vi) identify this particle.*

If this baryonic resonance has a kinetic energy of 200 GeV, how far will travel before decaying?

A few necessary numerical values are the following:

$$\hbar c = 197\,MeV \cdot fm\,; c = 3 \cdot \frac{10^8 m}{s}, m_{\Delta^{++¿}=1232\frac{MeV}{c^2}}, m_{\pi^{+¿}=140\,MeV/c^2¿¿}, m_p = 940\,MeV/c^2$$

Second option

*II.1. At the **R**elativistic **H**eavy **I**on **C**ollider (RHIC) from **B**rookhaven **N**ational **L**aboratory (BNL) nucleus-nucleus collisions at energies, in the center of mass system, between* $\sqrt{s_{NN}} = 7{,}7\,GeV$ *up to* $\sqrt{s_{NN}} = 200\,GeV$ *can been performed. The main experimental results have been obtained for Au-Au collisions, Cu-Cu collisions, d-Au collisions and p-p collisions.*

(a) Calculate the necessary time to pass one nucleus through the other nucleus

(b) If a human body is in the immediate proximity of the pipe beam, calculate the biological dose (equivalent dose) for the minimum and maximum energies for the four collision types

(c) In nucleus-nucleus collisions at these energies, new phase of the nuclear matter could appear. The most important is that of quark-gluon plasma. One of the experimental signals of this phase is the increase of strangeness, as well as the hypernuclei formation. In one of the statistical models proposed, the density of the particles of type "i" can be calculate with the fooling formula:

$$n_i = g_i . \frac{1}{2\pi^2} T\, m_i^2\, K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}}$$

(i) Specify the signification of the physical quantities included in this relationship.

(ii) In a statistical model the ratio of two hyperons that can contribute to the hypernuclei formation can be evaluated using the "freeze-out" temperatures. For a "freeze-out" temperature of 150 MeV and a strangeness chemical potential of 20 MeV, calculate the Ω^-/Ξ^- ratio.

The hyperon Ω^- has the following quarks structure (sss), and the hyperon Ξ^- has this quarks structure: (dss). Its rest masses are $m_{\Omega^-} = 1672\ MeV/c^2$, $m_{\Xi^-} = 1321\ MeV/c^2$, respectively.

At this energy, the Bessel function can be approximately as follows: $K_2(m/T) \sim exp(-m/T)$.

Third option

*III.1. At GSI Darmstadt is in final stage the construction of a new accelerators system, named **F**acility for **A**ntiproton and **I**on **R**esearch (FAIR). This system will use the existing linac and synchrotron SIS 18. Two new synchrotrons will be added, namely SIS 100 and SIS 300. Explain the significance of the figures/numbers existing after the common name SIS and explain the order of the magnitude of the energies that can be obtained with each of the three synchrotrons.*

At FAIR different nucleus-nucleus collisions, in the laboratory system, will be performed. Considering the very high interaction rate expected in Au-Au collisions (around 1 MHz), interesting information on the neutron star formation will be obtained.

In the Hubble scenario of the Universe evolution, entropy is an important physical quantity. In models proposed for describing the dynamics of the relativistic nuclear collisions the t/d ratio and ^{3}He/d are used for Shanonn entropy determination. In one of the statistical models proposed, the density of the particles of type "i" can be calculate with the fooling formula:

$$n_i = g_i . \frac{1}{2\pi^2} T\, m_i^2\, K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}}$$

(i) Specify the signification of the physical quantities included in this relationship.

(ii) Using this relationship, calculate the two ratios, in the following approximation for the Bessel function: $K_2(m/T) \sim exp(-m/T)$.

The "freeze-out" temperature expected for Au-Au collision at maximum SIS 100 energy is 100 MeV, and the baryonic chemical potential is around 350 MeV. The masses of the involved nuclei are: $m_d = 1{,}8756\ GeV/c^2$, $m_{He\text{-}3} = 2{,}8932\ GeV/c^2$, $m_t = 2{,}8089\ GeV/c^2$.